Journal of Engineering Mechanics

A unified model for geomaterial solid/fluid states and the transition in between --Manuscript Draft--

Manuscript Number:	EMENG-1560R2
Full Title:	A unified model for geomaterial solid/fluid states and the transition in between
Manuscript Region of Origin:	FRANCE
Article Type:	Technical Paper
Keywords:	Solid-fluid transition, failure, second-order work, elasto-plasticity, Bingham's model.
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Corresponding Author:	Noémie Prime 3S-R laboratory Grenoble Cedex 9, Isère FRANCE
Corresponding Author E-Mail:	noemie.prime@gmail.com;noeprime@hotmail.com;noemie.prime@3sr-grenoble.fr
Order of Authors:	Noémie Prime Frédéric Dufour Félix Darve
Suggested Reviewers:	<p>Manuel Pastor manuel.pastor@upm.es Pr. Pastor has a strong experience on landslide numerical modeling, both on the initiation and the propagation stage.</p> <p>Pierre Yves Hicher pierre-yves.hicher@ec-nantes.fr Pr. Hicher has a strong expertise in elaborated elasto-plastic and visco-plastic model for soils.</p> <p>Claudio di Prisco cdiprisc@stru.polimi.it Pr. di Prisco is very experienced on elaborated numerical models for soils, applied notably for landslide modeling.</p> <p>Richard Wan wan@ucalgary.ca Pr. Wan has a large experience in numerical modeling, notably in complex models for granular geomaterial, and in general for soils and rocks.</p>
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A unified model for geomaterial solid/fluid states and the transition in between

N. Prime¹, F. Dufour², and F. Darve³

Abstract: During mudflows, geomaterials evolve in a very specific manner as, initially behaving as solids, they turn to flow as viscous fluids. The present paper proposes an original approach to simulate – within a single numerical framework – this transition between a solid-like and a fluid-like behavior. The model used is based on the association of an elasto-plastic and a viscous constitutive relation to describe both phases of the behavior. The transition between the two is assumed to correspond to a failure state, and the transition criterion considered is thus based on the general second-order work stability criterion. Because of their suitability to describe respectively soils and granular suspensions, Plasol elasto-plastic model and Bingham’s viscous one have been considered. In this preliminary study, the global model is tested for two homogeneous loading paths with drained and undrained conditions. For these cases, it results that the radical transformation of the material behavior at failure is captured and both the solid-to-fluid and the fluid-to-solid transitions are consistently enabled, making possible a future application of this approach to gravitational flow modelling.

Keywords: Solid–fluid transition, failure, second-order work, elastoplasticity, Bingham’s model.

¹ Université de Liège. noemie.prime@gmail.com, Université de Liège, Dept. ArGEnCo, Chemin des Chevreuils 1, 4000 Liège, Belgium.

² Professor, Grenoble INP / Université Joseph Fourier / CNRS UMR 5521, 3SR laboratory.

³ Professor, Grenoble INP / Université Joseph Fourier / CNRS UMR 5521, 3SR laboratory.

18 **Introduction**

19 In-situ soils have been extensively studied by both the geomechanical and the geotechnical
20 communities and are now well known to exhibit non-associated elastoplastic behavior.
21 Furthermore, a plastic failure surface defines the limit stress states beyond which no static
22 equilibrium can be found. Elastoplastic behavior is thus classically used to describe the initiation of
23 landslides by reaching a failure state in a natural slope (for a stress state which can be situated on
24 the plastic limit criterion or inside this criterion, see section 2) (Lignon et al. 2009), but this failure
25 stage usually leads to a numerical divergence of the computations. Moreover, this divergence is
26 often considered an indicator for material instability (Klubertanz et al. 2009).

27 Nevertheless, during the triggering of flow-type landslides, it has been observed that the solid soil
28 can evolve into a mudflow of an amazing fluidity. For instance, landslides in Gansu (China, 7
29 August 2010) and in Sarno and Quindici (Italy, 5 May 1998), developed this way. These flows of
30 soils have been shown to be well fitted by a Herschel-Bulkley viscous behavior characterized by a
31 yield stress (Daido 1971) and non-linear viscosity (Coussot and Boyer 1995; Malet et al. 2004),
32 which is often characterized by a shear thinning due to the granular structure disorganization. This
33 type of yield stress viscous behavior has already been used to model the propagation of flowslides
34 (Pastor et al. 2009). Moreover, experiments on granular suspensions show that the solid–fluid
35 transition depends on the history of the system and on the energy supplied to it (Coussot et al.
36 2005). In dry granular materials, the drastic influence of loading on slide behavior is also studied by
37 physicists (Jaeger and Nagel 1996), which introduced the notion of ‘jamming’ to characterize that
38 this material behavior can be “solid like” –it is referred to as the jammed state- as the sand at rest, or
39 “fluid like” – it is referred to as the unjammed state- as sand avalanches.

40 Since the behavior of geomaterials can evolve from a solid-like type to a fluid-like type, a natural
41 question is: how can the solid part of the behavior, the fluid part, and the transition between the two
42 be described within a single framework?

43 For decades, great effort has been devoted to accounting for, as accurately as possible, the transition
44 between the two states in granular materials. First of all, in granular suspensions, some models are
45 defined with a purely viscous relation, in which the transition of the behavior is expressed by a
46 variation of viscosity between physical values up to values tending to infinity (inducing jamming),
47 depending on the conditions of the tests. For instance, Coussot et al. (2002) defined such a
48 thixotropic constitutive relation to reproduce the flowing and the jamming of bentonite suspensions.
49 But this approach always considers time-dependent behavior, even for the solid phase, which is
50 debatable. A second kind of constitutive relation defines a jamming stress criterion beyond which a
51 viscous flow is activated. Many models belong to this family; from very simple ones (e.g., the
52 Bingham model, Fig.1a) to greatly elaborated ones (e.g., the model proposed by Jop et al. (2006) for
53 dense granular flows). However, their main drawback is that, even if the flowing domain is bounded
54 by what can be considered a solid–fluid transition criterion, the solid behavior inside this surface (in
55 the stress space) is not defined. Finally, constitutive relations of the Perzyna type (Perzyna 1966)
56 overcome this shortcoming by introducing – in the 1D assumption – an elastic component in series
57 to a Bingham model (Fig. 1b). However, in the hardening regime, the Perzyna constitutive relation
58 leads to non-instantaneous plastic strains, and this is an open question.

59 To our knowledge, although geomaterials can alternatively behave as a solid or a fluid, no
60 constitutive relations are suitable to describe both phases and the transition between them.
61 Therefore, a new 3D unified model is proposed in this paper. Indeed, the main originality of this
62 model lies in two new developments: the association of suitable fluid and solid constitutive relations

63 and the use of a general criterion to detect the first solid–fluid transition, i.e., the second-order work
64 criterion.

65 This paper is organized as follows. In the first section, the second-order work criterion is described
66 as a candidate for the solid–fluid transition criterion. Then the elastoplastic constitutive relation
67 used for the pre-failure behavior is briefly reviewed, while the equations of the 3D Bingham model
68 are established. Finally, preliminary illustrations of the capabilities of the whole model are
69 presented together with parametric analyses. Let us note that, in this first step, only tests on
70 homogeneous single-phase materials are considered, but fully exhibiting solid-like elastoplastic
71 behavior before failure and fluid-like viscous behavior after failure. For that, drained and undrained
72 loading conditions on a proper soil are considered, leading to classical or non-classical failure.

73 **A model for the solid–fluid transition**

74 If failure is defined, in a conventional manner, as the existence of limit stress–strain states, then it is
75 always related to an unstable behavior. Indeed, at these limit states, if an infinitely small load
76 increment is applied, a large material response is induced.

77 Instabilities may be divided into two main classes: one related to specific boundary conditions (the
78 “geometric instabilities”) and the other to particular material properties and states (the “material
79 instabilities”). Geotechnical issues generally stem from material instabilities and therefore this
80 contribution focuses on this category. Material instabilities are themselves of two types: the
81 “divergence instabilities” where, in a nutshell, the strains suddenly increase monotonously to high
82 values, and the “flutter instabilities,” which correspond to cyclically varying strains of increasing
83 amplitude (Bigoni and Noselli 2001). The failures occurring in landslides are generally linked to
84 divergence instabilities. Finally, in this last group, various patterns can also be distinguished in situ,
85 such as localized failure by shear band formation (Desrues and Chambon 2002) or diffuse failure

86 (Daouadji et al. 2011). Regarding these different modes of failure for geomaterials, let us analyze
87 the existing theories for stability analysis.

88 *Elastoplastic limit theory*

89 Today, the most classical criterion to define the failure state is the plastic limit criterion. With
90 engineering notations (strain and stress tensors denoted by six component vectors), this is
91 formalized as follows: let $\mathbf{M}$ be the constitutive second-order tensor connecting the stress and strain
92 increments ($d\boldsymbol{\sigma} = \mathbf{M}d\boldsymbol{\varepsilon}$); the failure condition is: $\det(\mathbf{M})=0$. In other words, a bounded stress
93 increment no longer produces a bounded strain response, as expressed by the Lyapunov stability
94 definition (Lyapunov 1907), but an unlimited response. This failure mode is often called "classical
95 failure." This criterion has a general applicability for associated materials such as metals (Rice
96 1971). However, for non-associated materials such as soils, the plastic limit condition cannot
97 always predict the loss of stability for localized (Fig. 2a) or diffuse failures (Fig. 2b) (Nicot and
98 Darve 2011). For instance, a diffuse failure is typically obtained during an undrained triaxial test on
99 loose sand for which the material suddenly collapses at the peak of the deviatoric stress q (if the test
100 is driven by the axial force), although the plastic limit criterion has not yet been reached (Fig. 2b,
101 2c).

102 Thus, elastoplastic limit theory with stress limit states is not able to detect all types of instability in
103 geomaterials (Darve et al. 2004; Laouafa et al. 2002), which is a major drawback to accurately
104 ensuring safety conditions in the context of geotechnical work (excavation work, tunnelling, etc.) or
105 natural hazard prediction (landslides, mudflows, etc.).

106 *The second-order work criterion*

107 Given this limitation of the classical plastic limit theory, other theories have been developed to
108 predict failure. Firstly, localized failures have been characterized by the vanishing of the

109 determinant of the acoustic tensor, according to the Rice criterion (Rudnicki and Rice 1975). In
 110 addition, diffuse failures (Nicot et al. 2011) could be explained by the loss of definite positiveness
 111 of the elastoplastic matrix $\mathbf{M}$, as follows (Hill 1958):

$$112 \quad \det(\mathbf{M}_s) \leq 0, \quad (1)$$

113 where $\mathbf{M}_s$ is the symmetric part of $\mathbf{M}$. The vanishing of $\det(\mathbf{M}_s)$ defines a three-dimensional limit
 114 surface between a stability domain and a bifurcation domain (Prunier 2009) in the principal stress
 115 space. By introducing the so-called second-order work d^2W , a sufficient condition for stability
 116 becomes, at the solid scale, the following:

$$117 \quad d^2W = \int_V d\sigma_{ij} d\epsilon_{ij} dV > 0 \quad \forall d\epsilon \neq 0, \quad (2)$$

118 $d\sigma_{ij}$ being related to $d\epsilon_{ij}$ by the elastoplastic constitutive relation. V is the volume occupied by the
 119 solid studied, strains are assumed to be small and geometrical effects are ignored. For a material
 120 point, a local expression is defined (Laouafa and Darve 2002) and its normalized form is:

$$121 \quad d^2W_n = \frac{d\sigma_{ij} d\epsilon_{ij}}{|d\sigma||d\epsilon|} > 0 \quad \forall d\epsilon \neq 0, \quad (3)$$

122 $d\sigma_{ij}$ being related to $d\epsilon_{ij}$ by the elastoplastic constitutive relation. According to this expression,
 123 d^2W_n stands for the cosine of the angle between the stress and the strain increment directions.
 124 Thus, stability depends both on stress and strain paths, contrary to the plastic limit criterion where a
 125 given stress state is sufficient to determine whether failure is met. According to Darve et al. (2004),
 126 if condition (2) is not verified (i.e. at failure) it means that the system is still subjected to strains,
 127 while no more energy is transferred to it, i.e. "...the deformation process continues on its own."
 128 This notion can be related to the notion of "loss of controllability" (Nova 1994). More precisely,
 129 when the second-order work is taking negative values in at least one loading direction, a small

130 additional load in this direction – or more generally a perturbation – leads to a burst of kinetic
131 energy (Nicot et al. 2011) and to a transition from a quasi-static regime to a dynamic one (Nicot et
132 al. 2011a).

133 It is essential to underline that, according to linear algebra, the loss of definite positiveness of the
134 elastoplastic matrix (Eq. 1) includes, as particular cases, the plastic limit condition (vanishing of the
135 determinant of the elastoplastic matrix) and the localization condition (vanishing of the determinant
136 of the acoustic matrix) (Bigoni and Hueckel 1991; Nicot and Darve 2011). The particular case
137 where failure occurs on the plastic limit criterion corresponds to a non-zero strain increment for
138 which the stress increment is zero, and hence, so is d^2W_n . Furthermore, it has been shown more
139 recently that the second-order work criterion is able to describe any kind of divergence instability
140 for non-conservative elastic systems (Challamel et al. 2010; Lerbet et al. 2012). This is therefore the
141 general criterion of failure for rate-independent non-associated geomaterials (Daouadji et al. 2011).

142 The second-order work criterion has been successfully applied to geomechanical issues (Darve and
143 Laouafa 2000; Laouafa and Darve 2002; Lignon et al. 2009). It has also been extended to
144 unsaturated soils submitted to instabilities caused by hydraulic conditions (Buscarnera and di Prisco
145 2012). At the material scale, the power of the second-order work is typically highlighted by the
146 undrained triaxial test on loose sand for which d^2W_n changes sign at the deviatoric stress peak,
147 without reaching the plastic limit criterion (Fig. 2c).

148 The second-order work is therefore the most general failure criterion for soils, and most particularly
149 it can detect diffuse failure modes for which the soil is deeply and globally destructured (burst of
150 kinetic energy, possible liquefaction, etc.). For these reasons, it has been naturally chosen to detect
151 the solid–fluid transition state.

152 ***General form of the newly proposed model***

153 Based on the second-order work transition criterion, the model takes into account a solid-like pre-
154 failure behavior (with non-associated elastoplasticity) and a fluid-like post-failure behavior (with
155 viscosity and a yield stress). The model is formulated such that total strains are the sum of
156 elastoplastic and viscous strains, if any (Fig. 3). The latter are nil before the failure and are activated
157 as soon as $d^2W_n \leq \varepsilon$ (and if the yield stress is exceeded). Viscous strain rate activation is thus the
158 only simple way the transition is considered. A limit value of $\varepsilon=10^{-6}$ is arbitrarily chosen for d^2W_n
159 to activate the transition even if the limit load is obtained asymptotically.

160 The global consistency is ensured by the fact that the second-order work transition criterion remains
161 the same for any rate-independent constitutive relations. The dissociation of the two parts of the
162 strains makes it possible to plug in any solid or fluid model independently. The model is thus
163 adjustable with respect to any recent developments made in solid or fluid mechanics and to a wide
164 range of materials.

165 **Solid and fluid constitutive relations**

166 ***A pre-failure behavior: the Plasol elastoplastic model***

167 The elastoplastic model called Plasol (Barnichon 1998) is chosen since it reproduces the main
168 features of soils:

169 - Firstly, it is based on a Van Eekelen plastic criterion (Van Eekelen 1980), which is close to the
170 Mohr-Coulomb plastic criterion without geometric singularities. It is expressed as:

$$171 \quad F = J_{2\sigma} + m \left(J_{1\sigma} - \frac{3c}{\tan \varphi_c} \right) = 0 \quad (4)$$

$$172 \quad \text{with : } m=a (1+b \sin 3\theta)^n \quad \text{and : } \sin 3\theta= -\sqrt{6}\left(\frac{J_{3\sigma}}{J_{2\sigma}}\right)^3$$

$$173 \quad \text{where } b=\frac{(r_c/r_e)^{1/n}-1}{(r_c/r_e)^{1/n}+1}, \quad a=\frac{r_c}{(1+b)^n}, \quad r_c=\frac{1}{\sqrt{3}}\left(\frac{2\sin\varphi_c}{3-\sin\varphi_c}\right), \quad r_e=\frac{1}{\sqrt{3}}\left(\frac{2\sin\varphi_e}{3-\sin\varphi_e}\right)$$

174 $J_{1\sigma}=tr(\boldsymbol{\sigma})$, $J_{2\sigma}=\sqrt{tr(s^2)}$ and $J_{3\sigma}=\sqrt[3]{tr(s^3)}$ are the three invariants of the Cauchy stress tensor $\boldsymbol{\sigma}$
 175 ($s=\boldsymbol{\sigma}-1/3tr(\boldsymbol{\sigma})\mathbf{I}$ is the deviatoric part of $\boldsymbol{\sigma}$) and c is the material cohesion. a and b depend on the
 176 material friction angles in compression and extension, φ_c and φ_e respectively. These two angles
 177 together with the Lode angle θ modify the shape of the criterion in the deviatoric stress plane and
 178 determine a non-constant radius of the trace (Fig. 6b), contrary to a Drucker-Prager criterion. n is a
 179 dimensionless parameter whose value of -0.229 has been calibrated by Barnichon (1998) to ensure
 180 the convexity of the yield surface.

181 - Secondly, a hardening regime is described. The plastic parameters vary – according to the
 182 equivalent plastic strain $E_{eq}^p = \sqrt{2/3}e_{ij}^pe_{ij}^p$ (with $\mathbf{e}^p = \boldsymbol{\varepsilon}^p - 1/3tr\boldsymbol{\varepsilon}^p.\mathbf{I}$ being the deviatoric part of plastic
 183 strain tensor) and two parameters B_φ and B_c – between elastic limit values ($c_o, \varphi_{eo}, \varphi_{co}$) and plastic
 184 limit values ($c_f, \varphi_{ef}, \varphi_{cf}$), such as:

$$185 \quad \left\{ \begin{array}{l} \varphi_c = \varphi_{c0} + \frac{(\varphi_{cf} - \varphi_{c0})E_{eq}^p}{B_\varphi + E_{eq}^p} \\ \varphi_e = \varphi_{e0} + \frac{(\varphi_{ef} - \varphi_{e0})E_{eq}^p}{B_\varphi + E_{eq}^p} \\ c = c_0 + \frac{(c_f - c_0)E_{eq}^p}{B_c + E_{eq}^p} \end{array} \right. \quad (5)$$

188 Using Eq. 4, ($c_o, \varphi_{eo}, \varphi_{co}$) and ($c_f, \varphi_{ef}, \varphi_{cf}$), respectively, define the elastic limit and the plastic failure
 189 criterion. In all applications of the present study, the final values of c and φ are set strictly larger
 190 than the initial values in order to avoid softening and associated mesh dependencies. In the global

191 model, the plastic limit condition is included in the more general second-order work failure
192 criterion.

193 - Finally, Plasol can describe a non-associated plastic flow, i.e. the incremental plastic strain vector
194 is not perpendicular to the surface defined by F (Eq. 4) but instead to the plastic potential surface,
195 whose equation is:

$$196 \quad G = J_{2\sigma} + m' \left(J_{1\sigma} - \frac{3c}{\tan \varphi_c} \right) = 0 \quad , \quad (6)$$

197 where m' is defined as m (see Eq. 4) substituting the friction angles with the dilatancy angles, ψ_c in
198 compression and ψ_e in extension. Dilatancy angles vary with the same amplitude as friction angles,
199 according to the Taylor rule (1948): $\varphi_{cf} - \varphi_c = \psi_{cf} - \psi_c$. Consequently, only the final values of ψ need be
200 given.

201 From an experimental point of view, triaxial tests can directly determine cohesion and friction
202 angles from the failure envelope in the stress diagram, and dilatancy angles from the volume
203 variation monitoring. In contrast, B_c and B_φ have to be calibrated according to these triaxial results.

204 ***A post-failure behavior: 3D Bingham viscosity***

205 The fluid behavior of post-failure soils is chosen to reproduce the rheology of mudflows.

206 *Main features of mudflows*

207 Mudflows are known to follow a Herschel-Bulkley model (Fig. 4). However, given that the viscous
208 nonlinearity is often quite insignificant, the behavior can be considered in a first approximation with
209 a Bingham constitutive relation (i.e., linear viscosity beyond a yield stress).

210 *Inversion of the 3D Bingham constitutive relation*

211 In the context of a global solid–fluid transition model where elastoplasticity is defined in 3D, the
 212 Bingham constitutive relation must also be expressed in 3D. Moreover, the formulation of the
 213 model (the total strain rate tensor is the sum of the elastoplastic and the viscous components)
 214 requires to write the strain rate tensor. The key point is to determine in a consistent manner the
 215 stress threshold direction in 3D.

216 On one hand, as is well known, the Bingham 1D relation for positive or negative shearing is (Fig.
 217 5a):

$$218 \quad \text{if } \dot{\gamma} \neq 0: \quad \tau = \eta \dot{\gamma} + s_0 \cdot \text{sgn}(\dot{\gamma}), \quad \text{else: } |\tau| \leq s_0, \quad (7)$$

219 with η the dynamic viscosity, τ the shear stress, $\dot{\gamma}$ the velocity gradient and s_0 the stress threshold.

220 The function $x \rightarrow \text{sgn}(x)$ returns the sign of the scalar x . Since τ in Eq. 7 is a one-to-one relation on
 221 $]-\infty; 0[\cup]0; +\infty[$, it is invertible on this interval and its inverse function is (Fig. 5b):

$$222 \quad \text{if } |\tau| > s_0: \quad \dot{\gamma} = (\tau - s_0 \cdot \text{sgn}(\tau)) / \eta, \quad \text{else: } \dot{\gamma} = 0 \quad (8)$$

223 The sign of the stress threshold is here given by the sign of the shear stress.

224 On the other hand, the 3D Bingham model is usually expressed such that the deviatoric stress tensor
 225 depends on the deviatoric strain rate tensor. Various authors (Duvaut and Lions 1971; Balmforth
 226 and Craster 1999) define the stress threshold direction as the deviatoric strain rate tensor direction
 227 $(\dot{e}_{ij} / J_{2\dot{e}})$:

$$228 \quad \text{if } J_{2\dot{e}} \neq 0: \quad s_{ij} = 2\eta \dot{e}_{ij} + s_0 \frac{\dot{e}_{ij}}{J_{2\dot{e}}}, \quad \text{else: } J_{2\sigma} \leq s_0, \quad (9)$$

229 where $\mathbf{s}$ and $\dot{\mathbf{e}}$ are the deviatoric part of the stress and the strain rate tensors, $J_{2\sigma}$ and $J_{2\dot{e}}$ the
 230 second invariants of the stress tensor and the strain rate tensor. According to the 1D inversion, one

231 can consider that the 3D relation is also invertible, and the stress threshold direction is provided by
 232 the stress tensor direction ($\mathbf{s} / J_{2\sigma}$):

$$233 \quad \text{if } J_{2\sigma} > s_0: \quad \dot{\epsilon}_{ij} = \frac{1}{2\eta} \left(s_{ij} - s_0 \frac{s_{ij}}{J_{2\sigma}} \right) = \frac{J_{2\sigma} - s_0}{2\eta} \cdot \frac{s_{ij}}{J_{2\sigma}}, \quad \text{else: } \dot{\epsilon}_{ij} = 0 \quad (10)$$

234 η and s_0 of a mud suspension can be determined by inclined plane tests with different slope angles
 235 or by rheometer tests with different strain rates applied (see Coussot and Boyer 1995). s_0 therefore
 236 has no relation with the failure stress state.

237 At the end, considering both Plasol and Bingham's relations, the global model needs 15 parameters
 238 to be identified: two related to elasticity (E and ν), 11 to plasticity (φ_{co} , φ_{cf} , φ_{eo} , φ_{ef} , c_o , c_f , ψ_c , ψ_e , B_φ ,
 239 B_c , n), and two to viscosity (η and s_0).

240 *Synthesis of transition management*

241 With an initial elastoplastic material, the solid-to-fluid transition takes place when a failure state is
 242 reached. As the viscous strain rates develop only if the stress threshold is exceeded, this condition
 243 therefore also needs to be satisfied to enable the transition. Nevertheless, Fig. 6 shows that, for
 244 realistic parameters, the stress domain where the failure is possible ($J_{2\sigma}$ belonging to the plastic
 245 domain, between the elastic limit and the plastic limit state) is included in the domain where the
 246 viscous flow is possible ($J_{2\sigma} > s_0$). Reaching failure would thus be, in general, sufficient to induce
 247 an effective solid-to-fluid transition.

248 In the fluid phase, the behavior is still viscoelastoplastic, but the order of magnitude of viscosity is
 249 so low with respect to elastic parameters (see 4.1.b) that the deviatoric part of the stresses are
 250 quickly relaxed and the behavior is almost viscous. If $J_{2\sigma}$ becomes smaller than s_0 , the viscous

251 strain rates are disabled (according to Bingham model) and the material becomes elastoplastic
252 again.

253 **Analysis of the model on homogeneous tests**

254 Two tests are simulated hereafter with the aim of testing the constitutive model both at the material
255 point level and at a continuum scale. For the latter task, the finite element code Ellipsis based on the
256 advanced Finite Element Method with Lagrangian Integration Points (FEMLIP, Moresi et al. 2003)
257 was used. As in a standard Eulerian FE code in fluid mechanics, the domain of study is discretized
258 by means of a fixed computational grid on which nodal unknowns of the problem are computed
259 through an integration scheme over elements using integration points (that would be Gaussian
260 points in standard methods). Contrary to standard FE methods, the material domain is discretized by
261 a set of Lagrangian points (particles) carrying all material properties and variables, including
262 history-dependent variables. In a given material configuration, these points are used as integration
263 points, instead of Gaussian points, after being remapped to the natural element they belong to. At
264 each step, the numerical weight of each integration point needs to be updated in order to satisfy the
265 conditions of the Gaussian quadrature, such that the integration is the most accurate possible for the
266 aimed polynomial degree (Moresi et al. 2003). Once the nodal velocity field is computed, the
267 particle velocity is interpolated from the nodes with FE shape functions and the particles are moved
268 accordingly. The fixed computational grid confers no limit in deformation magnitude while the
269 moving Lagrangian particles make possible to track internal variables involved in complex
270 behaviors. Ellipsis has already been successfully applied, for example, to geophysical viscous
271 convection problems in 3D (O’Neil et al. 2006), viscoelastic convection (Moresi et al. 2003),
272 viscoelastic folding problems (Mühlhaus et al. 2002.a, 2002.b) and concrete flow in slump tests
273 (Dufour and Pijaudier-Cabot 2005; Roussel et al. 2007). This numerical tool has also been recently

274 used to analyze natural slope stability based on an elastoplastic constitutive model (Cuomo et al.
275 2012).

276 Drained and undrained triaxial tests, which can lead to different failure patterns, are considered
277 here. Since a drained triaxial test may exhibit a strain localization (Fig. 2a), it is better to model it at
278 the material point level to prevent the loss of homogeneity of the strain field (although Cosserat
279 regularization is implemented in Ellipsis (Mühlhaus et al. 2002.a)), whereas the undrained test is
280 simulated with Ellipsis, considering the plane strain conditions imposed by the code. A parametric
281 analysis is led on the viscous parameters in the undrained case.

282 ***The undrained loading path***

283 *Implementation of viscoelastoplasticity in Ellipsis*

284 In Ellipsis, Stokes' equations are solved and nodal unknowns are velocities and pressure. Thus,
285 the resolution matrix is made with viscous parameters. Since Ellipsis is based on a mixed
286 formulation to deal with incompressible materials, all tensorial equations are split into deviatoric
287 and isotropic parts. For better clarity, only the deviatoric equations are developed hereafter (see
288 Cuomo et al. 2012 for details).

289 First, the deviatoric strain rate is the sum of the viscous, elastic and plastic strain rates, which
290 gives:

$$291 \quad \dot{\mathbf{e}}_{tot} - \dot{\mathbf{e}}_{pl} = \dot{\mathbf{e}}_e + \dot{\mathbf{e}}_v = \frac{\tilde{\mathbf{s}}}{2\mu} + \frac{\mathbf{s}}{2\eta} \quad (11)$$

292 with index '*tot*', '*v*', '*e*' and '*pl*' respectively referring to the total tensor, and its viscous, elastic and
293 plastic parts. $\tilde{\mathbf{s}}$ is the Jaumann derivative of the deviatoric stress tensor $\mathbf{s}$ and μ is the Lamé shear
294 coefficient.

295 The time discretization of $\tilde{\mathbf{s}}$ on a time step δt_e makes the rotation tensor $\boldsymbol{\omega}$ appear, giving:

$$s^{t+\delta t_e} = 2\eta_{eff}(\dot{\epsilon}_{tot}^{t+\delta t_e} - \dot{\epsilon}_{pl}^{t+\delta t_e}) + \eta_{eff} \frac{s^t}{\mu \delta t_e} + \eta_{eff} \frac{\omega^t \cdot s^t - s^t \cdot \omega^t}{\mu} \quad (12)$$

The effective viscosity $\eta_{eff} = \eta \frac{\delta t_e}{\delta t_e + T}$ depends on the relaxation time $T = \eta/\mu$. For the isotropic behavior, the effective viscous bulk modulus is: $K_{v,eff} = K_v \frac{\delta t_e}{\delta t_e + T}$ assuming the same relaxation time as for the deviatoric part.

Developing the mechanical equilibrium equation ($\nabla \cdot s + \nabla p + f_{ext} = 0$) gives, using Eq. (12):

$$2\eta_{eff} \nabla \cdot (\dot{\epsilon}_{tot}^{t+\delta t_e} - \dot{\epsilon}_{pl}^{t+\delta t_e}) + \eta_{eff} \left[\frac{\nabla \cdot s^t}{\mu \delta t_e} + \frac{\nabla \cdot (\omega^t s^t - s^t \omega^t)}{\mu} \right] + K_{v,eff} \nabla \left[tr(\dot{\epsilon}_{tot}^{t+\delta t_e} - \dot{\epsilon}_{pl}^{t+\delta t_e}) + \frac{p^t}{K \delta t_e} \right] + f_{ext}^{t+\delta t_e} = 0 \quad (13)$$

where K is the elastic bulk modulus. Gathering all the terms from the previous time step t in an elastic force term coming from the stored stress tensor, and all the terms depending on the plastic response in a plastic force term, gives for the current time $t + \delta t_e$:

$$2\eta_{eff} \nabla \cdot \dot{\epsilon}_{tot}^{t+\delta t_e} + K_{v,eff} \nabla tr(\dot{\epsilon}_{tot}^{t+\delta t_e}) + f_{ext}^{t+\delta t_e} + f_{elastic}^t + f_{plastic}^{t+\delta t_e} = 0 \quad (14)$$

The force terms are updated at each time increment. However, loading is such that plastic strains are supposed not to vary much between two steps and, at a given time $t + \delta t_e$, $f_{plastic}^{t+\delta t_e}$ is replaced with the known quantity $f_{plastic}^t$. As the plastic force term is explicit, different magnitudes of the loading increments have to be tested in order to verify that the solution does not depend on it. Besides, pure elastic or elastoplastic behaviors can be addressed in the limit by setting a very high value to the viscosity η which becomes a numerical parameter. This way, η_{eff} can be defined but almost no relaxation occurs along the time steps.

To describe the solid-to-fluid transition process, the right hand side of equation (11) takes into account, before the transition, one elastic and one ‘numerical’ viscous term. In a second time, once

the geomaterial fulfills the conditions for the flow ($d^2W_n \leq \varepsilon$ and $J_{2\sigma} > s_0$), an additional viscous strain rate term is considered in eq. (11) such as:

$$\dot{\mathbf{e}}_{tot} - \dot{\mathbf{e}}_{pl} = \dot{\mathbf{e}}_e + (\dot{\mathbf{e}}_v)_{num} + (\dot{\mathbf{e}}_v)_{phy} = \frac{\tilde{s}}{2\mu} + \frac{s}{2\eta_{num}} + \frac{J_{2\sigma} - s_0}{2\eta_{phy}} \cdot \frac{s}{J_{2\sigma}} \quad (15)$$

In this equation, the expression of $(\dot{\mathbf{e}}_v)_{phy}$ corresponds to the physical behavior of the geomaterial flow, and it is expressed according to eq. (10). This new term induces an additional force term in the formulation of the problem (see eq(14)) equal to $\mathbf{f}^t = -(\eta_{eff} / \eta_{phy}) s_0 \nabla \cdot (s/J_{2\sigma})^t$, and a

modified expression of the effective viscosity such as: $\eta_{eff} = \eta_{num} \frac{\delta t_e}{\delta t_e + T + \delta t_e (\eta_{num} / \eta_{phy})}$.

Description of the model

The different behaviors being accounted for in Ellipsis, a unit square representative elementary volume of soil is considered with the plane strain conditions of the code. The mechanical parameters, gathered in Table 1, are chosen to be representative of a clayey silt before failure (low elastic parameters, quite high friction angle and cohesion). The friction and the dilatancy angles in compression are the same as the friction and the dilatancy angles in extension. Relating to flowslide hazards, a broad scattering for rheological parameters is found in the literature for natural events themselves (Jeyapalan et al. 1983; Pastor et al. 2008; Soga 2011) or for artificial mud mixtures (Coussot and Boyer 1995; Coussot et al. 1996), with viscosity ranging from 50 to 1000 Pa.s and the stress threshold ranging from 100 to 5000 Pa, depending on the soil characteristics. In this preliminary analysis, low values of s_0 and η are arbitrarily chosen (100 Pa and 50 Pa.s, respectively). The material has an initial pressure (that is to say a mean stress) of 30 kPa. Compression strain in vertical direction z and extension strain in horizontal direction x are both increasingly applied to the square, so as to keep a constant volume ($\dot{\varepsilon}_{zz} = -\dot{\varepsilon}_{xx} = 0.6 s^{-1}$). The strain rate conditions are considered according to the Ellipsis viscous formulation, although time has no

physical meaning until the transition. Even if the hydraulic phase is not accounted for in the present model, this isochoric loading has the same effect on the sample as undrained conditions on a saturated material due to the quasi-incompressibility of water and solid grains. d^2W_n is calculated for each material point and at each step, according to Eq. 3, but considering only the current loading direction. Stress and strain increments are thus given by the difference between the tensors for the current and the previous time steps.

Results and analysis

The second stress invariant $J_{2\sigma}$ is plotted as a function of pressure and physical time (Fig. 7 and Fig. 8, respectively), the pressure axis being better suited to studying the solid-like response before failure, and the time axis better adapted to describing the fluid-like response.

Once the elastic limit is reached (point 'Q' in the two graphs), the pressure consistently decreases (points 'Q' to 'R') since the soil is initially contractant at this stage (the mobilized dilatancy angle is $\psi_o = \psi - (\phi_f - \phi_o) = -20^\circ$ according to Plasol model) and the isochoric conditions prevent the sample from contracting.

Concerning the transition, the expression of the second-order work for such an undrained test is:

$d^2W_n = dq \cdot d\varepsilon_{zz}$ with $dq = d\sigma_{zz} - d\sigma_{xx}$. For a linear increase of ε_{zz} , failure is thus reached at the peak of q , which corresponds here to the peak of $J_{2\sigma}$ (Fig. 7 point 'R'). At this step, $J_{2\sigma}$ suddenly decreases and the stress state turns back into the elastic domain ('R' to 'S'). Accordingly, the computation shows that no plastic strain occurs and the pressure remains constant after failure (Fig. 7), i.e., no viscous volumetric strains can take place since the Bingham relation is purely deviatoric, and elastic volumetric strains are null because the loading is isochoric.

The time evolution of $J_{2\sigma}$ for such a viscoelastic material subjected to an axial strain rate is the solution of a partial differential equation and it is expressed as a function of physical time t :

$$J_{2\sigma} = (J_{2\sigma} - 2\eta\dot{\epsilon}_{zz} - s_o) \exp\left(\frac{-\mu}{\eta}t\right) + 2\eta\dot{\epsilon}_{zz} + s_o, \quad (16)$$

where $J_{2\sigma}$ is the second stress invariant at failure. Along the time axis (Fig. 8), $J_{2\sigma}$ after failure indeed coincides with this exponential analytical solution. When time tends to infinity, $J_{2\sigma}$ is defined by a pure Bingham relation, with one fraction of the stress induced by the constant strain rate loading and the other due to the yield stress s_o , which cannot be relaxed.

This computation describes a sudden change of the material behavior at failure with a relaxation of the deviatoric part of the stress tensor, which cannot indeed be sustained by fluids (unless $J_{2\sigma}$ is smaller than s_o). It can be observed in Fig.7 that the stress path obtained differs from the undrained elastoplastic path presented ('EP reference' curve). Indeed, it must be underlined in the results that, although the stress state before the transition does correspond to the inter-granular stresses within the soil, the stress state after the transition indeed corresponds to the stresses within a homogenized material (the granular suspension) which stands for the water-grain mixture. This is a consequence of the absence of coupling between two phases and it involves that, after the transition, the reference curve of Fig.7 and the calculated stress path cannot be compared in a rigorous manner. Besides, it has to be noticed that, for such an undrained test driven by a strain loading, the failure cannot develop due to the kinematic constraints, which is not the case if the test is stress controlled. The disorganization of the material thus appears to be progressive during the loading. Due to the absence of coupling in our approach, the transition has to be instantaneous, whatever the loading conditions. Nonetheless, the triggering of mudflows in the field is generally linked to mixed loading conditions (stress and displacement ones) with an important role of the stress conditions (rising of the water table level for instance). The total constraint of the kinematic, which prevents the effective failure development, is thus unlikely.

382 ***Parametric analysis***

383 Two parametric studies are conducted to evaluate the performance of the transitional model through
384 the large realistic range of viscosity and Bingham's threshold found in the literature for mudflows.
385 First, for a fixed $S_0 = 100$ Pa, five values of η are chosen: 50, 100, 300, 600 and 1000 Pa.s. The time
386 evolution of the second stress invariant $J_{2\sigma}$ is presented in Fig. 9a. As predicted by the analytical
387 solution (Eq. 16), the increase of viscosity leads to a higher final stress and a slower relaxation of
388 stresses (less decreasing exponential).

389 Then, for $\eta = 50$ Pa.s, four tests are performed with $S_0 = 100, 300, 2400$ Pa and $S_0 = 4800$ Pa (i.e.
390 defined as the second stress invariant at failure $J_{2\sigma_f}$). Results are presented in Fig. 9b. In the last
391 case, viscosity does not develop since the Bingham model cannot induce a viscous strain rate if
392 there is no difference between the second stress invariant and the stress threshold (Eq. 10). It would
393 be the same for any S_0 larger than $J_{2\sigma_f}$. The other cases are also consistent with Eq. 16 since the
394 higher S_0 , the higher the horizontal asymptote of $J_{2\sigma}(t)$.

395 ***The drained triaxial path***

396 In this second test, the mechanical parameters are the same as for the undrained test. A stress path is
397 followed with an initial confinement of $p_o = 100$ kPa (state marked as 'A' in all figures), an increase
398 of the axial deviatoric stress $q = \sigma_{zz} - p_o$ up to the failure (marked as 'B'), a constant stress state to let
399 the viscous strains develop (from 'B' to 'C'), and finally a decrease of q up to 0 (point 'D') to
400 highlight the jamming of the flow. The evolution of q is plotted along computational time (Fig. 10a)
401 and pressure (Fig. 10b).

402 During the first loading stage (A to B), the total axial strain ε_{zz} increases in a classical elastoplastic
 403 way (Fig.11), while the viscous component of ε_{zz} remains null (Fig.12c). The second-order work
 404 first presents a slight discontinuity at the elastic limit, then goes through a maximum before sharply
 405 decreasing when approaching the plastic limit criterion, reaching the limit value of 10^{-6} (B) (Fig.
 406 12a and 12b). It thus coincides, in this case, with the plastic limit criterion. It is worth noting in Fig.
 407 12c that, at failure, ε_{zz} evolves continuously over time, with a continuous derivative (no singular
 408 points). From this point, the viscous axial strain is activated according to the transition model and,
 409 since the axial stress is kept constant, ε_{zz} linearly increases (B to C in Fig. 12c).
 410 During the last stage (C to D), the application of a linear decrease of q up to 0 induces a linear
 411 decrease of $\dot{\varepsilon}_{zz}$. The flow finally stops (Fig. 13b) for a value of the second stress invariant which is
 412 not zero but 100 Pa, i.e. equal to S_0 (Fig. 13a). The model thus describes Bingham's jamming well.

413

414 **Conclusions and perspectives**

415 The 3D transitional constitutive model presented here has been formulated to describe, within a
 416 unified framework, both the elastoplastic behavior of an in-situ soil and its transition toward a fluid-
 417 like behavior, which occurs for example during mudflow triggering. The model is based on a
 418 transition criterion defined as the second-order work failure criterion, and therefore it is able to
 419 detect all types of divergence instabilities and is independent of the elastoplastic constitutive
 420 relation used. It also links adaptable solid and fluid constitutive models. In the second step, if a
 421 viscous relation with a yield stress is chosen, the arrest of the flow (i.e., the fluid-to-solid
 422 transition) is also described. A Plasol elastoplastic model and a Bingham viscous model are first
 423 considered, with an inverted expression of the 3D Bingham relation with respect to usual
 424 formulations. The two tests performed with this behavior validate the consistency of the model for

both drained and undrained conditions, and not only at the material point scale, but also implemented in a continuum finite element code. The drained triaxial test more specifically shows the consistency of the failure detection as well as the jamming of the flow, and it highlights the continuity of the total strains at the solid–fluid transition. The undrained test makes possible to verify the post-failure flow according to the viscoelastic analytical response. In the latter case, the model describes a drastic transition from an elastoplastic behavior towards a viscous one, although elastoplasticity still coexists in the model during the flow. This could describe the sudden collapse of soils observed during such a test, when it is axially force controlled.

The parametric analysis shows that Bingham's threshold needs to be lower than the second stress invariant at failure to enable the transition. Indeed, in the field, it is consistent to consider that the yield stress in mudflows is lower than the in-situ soil stress invariant at failure. This condition can be viewed as a reduction of the mechanical strength of a deeply remoulded soil.

In conclusion, due to the transitional behavior it describes, this model, now correctly implemented into a numerical code, would be very efficient in simulating gravitational flows, such as landslides of the flow type (mudflows, debris flows). The next step will thus be to apply it, with Ellipsis, to a heuristic unstable slope evolving into flow at failure. Besides, further developments are already underway to account for hydro-mechanical coupling to model partially saturated geomaterials.

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539

a) b)

Figure 1: The Bingham (a) and Perzyna (b) models in 1D

a) b) c)

Figure 2: Localized failure in a drained triaxial test (a) (Desrues and Chambon 2002), diffuse failure in an undrained triaxial (b) (Servant et al. 2005). Stress path for the undrained test (c) (Darve et al. 2004)

Figure 3: Diagram of the global constitutive model in 1D in a general form

Figure 4: Low nonlinearity in the Hershel Bulkley relation (Coussot and Boyer 1995)

Figure 5: Bingham one-to-one relation (a) and inverted relation (b) in 1D

a) b)

Figure 6: Characteristic surfaces of the global model for Plasol and Bingham relations ($c=3-10$ kPa, $\varphi_c=6-28$, $s_0=2$ kPa) (a) Trace of the surfaces in the deviatoric plane (b) (VE, Van Eekelen)

Figure 7: Stress path with regard to the elastoplastic (EP) case with no transition. For the transition curve: PQ= elastic response; QR= elastoplastic response; RS= viscoelastoplastic response.

Figure 8: $J_{2\sigma}$ over physical time and comparison with the analytical solution. For the transition curve: PQ= elastic response; QR= elastopalstic response; RS= viscoelastoplastic response.

564 **Figure 9:** Influence of viscosity (a) and yield stress (b)

565

566 **Figure 10:** Stress loading path along time (a) and pressure (b)

567

568 **Figure 11:** Deviatoric stress q vs axial strain ε_{zz}

569

570 **Figure 12:** d^2W_n over time (a), zoom (b) Onset of the solid–fluid transition (c)

571

572 **Figure 13:** Arrest of the flow (b) upon reaching Bingham yield stress (a)

573

574 **Table 1:** Constitutive parameters considered

ρ (kg/m ³)	E (MPa)	ν	φ_o / φ_f (°)	ψ (°)	c_o / c_f (kPa)	B_φ	B_c	n	η (Pa.s)	S_0 (Pa)
0	15	0.29	3 / 28	+5	1 / 10	0.01	0.02	−0.229	50	100

575

Figure s_0

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Figure

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Elasto-plasticity

```
graph LR; A[Elasto-plasticity] --> B(( )); B --> C{d^2W}; B --> D[Viscosity]; C --> E(( )); D --> E; E --> F[ ]; style A fill:#d3d3d3,stroke:#000,stroke-width:1px; style C fill:#808080,stroke:#000,stroke-width:1px; style D fill:#e0e0e0,stroke:#000,stroke-width:1px;
```

d^2W

Viscosity

Figure 3

[Click here to download Figure Fig4.pdf](#)

Shear stress (Pa)

Kaolin-water mixture (1):

● Parallel plates (Radii: 2.5 cm; Gap: 3 mm)

○ Holed cone-plate (Radii: 3 and 1.5 cm; Angle: 5.7°)

Sinard clay-water mixture (c):

□ Holed cone-plate

■ Parallel plates

— Herschel-Bulkley model

10³
10²
10¹
10⁻² 10⁻¹ 10⁰ 10¹ 10²

Shear rate (1/s)

Figure

[Click here to download Figure: Fig5.pdf](#)

Figure

[Click here to download Figure: Fig6a.pdf](#)

Figure

[Click here to download Figure: Fig6b.](#)

Figure

[Click here to download Figure: Fig7.pdf](#)

Figure 8

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Figure 6
[Click here to download Figure: Fig9.pdf](#)

Figure 10
[Click here to download Figure: Fig10.pdf](#)

Figure 250

[Click here to download Figure: Fig14.pdf](#)

Figure 6

[Click here to download Figure: Fig12.pdf](#)

a)

b)

c)

Figure

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SAMPLE PERMISSION REQUEST

Date of Request: November 18th 2012

To:

Philippe Coussot

Person (if known)

Research director and Manager of Physics of
Porous Media Team

Title (if known)

Institut Français des Sciences et Technologies
des Transports, de l'Aménagement et des
Réseaux (Ifsttar)

Company

Cité Descartes, 2 allée Képler

Address

77420 Champs-sur-Marne, France

Address

0 (033) 1 40 43 54 41

Telephone

0 (033) 1 40 43 54 50

Fax

philippe.coussot@ifsttar.fr

E-mail

From:

N. Prime – F. Dufour – F. Darve

Author

3SR laboratory

Affiliation

Domaine Universitaire, BP53,

Address

38041 Grenoble Cedex 9, France

Address

Address

0 (033) 4 56 52 86 26

Telephone

0 (033) 4 76 82 70 43

Fax

noemie.prime@3sr-grenoble.fr, noemie.prime@gmail.com

E-mail

I am preparing a manuscript to be published by the American Society of Civil Engineers, a not-for-profit corporation located in Reston, Virginia, in:

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with an estimated publication date of

April-March 2013

Estimated publication date

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534

543

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26 November 2012

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Jacques Desrues, manager of :

3SR laboratory
Grenoble INP / Université Joseph Fourier / CNRS UMR 5521
Domaine Universitaire
BP53
38041 Grenoble Cedex 9
France
Tél. : 0 (033) 4 76 82 51 73
Fax : 0 (033) 4 76 82 70 43

the:

November 18th 2012

by:

Noémie Prime
PhD student, Grenoble INP / Université Joseph Fourier / CNRS UMR 5521, 3SR laboratory
noemie.prime@3sr-grenoble.fr , noemie.prime@gmail.com , noeprime@hotmail.com
Laboratoire 3S-R, Domaine Universitaire, BP53, 38041 Grenoble Cedex 9, France
Tél. : 0 (033) 4 56 52 86 26
Fax : 0 (033) 4 76 82 70 43

for:

a publication (numerical and in printed version) in Journal of Engineering Mechanics (ASCE),
co-written by N. Prime, F. Dufour, F. Darve, which publication date could be estimated in
march-april 2013.

The figure which will be used:

in Figure 2a of our article : *Localized failure in a drained triaxial test.*

This pictures can be used for publication in ASCE Journal of Engineering Mechanics by N. Prime and co-authors.

The original picture comes from J.L. Colliat-Dangus Doctoral Thesis, Grenoble (France) 1986. I was co-advisor of this PhD.

Colliat-Dangus, J.L., 1986. Comportement des matériaux granulaires sous fortes contraintes. Ph.D. Thesis, USMG—INPG Grenoble.

The picture appeared first in the international literature as an illustration in the reference :

[Desrues J., Chambon R. \(2002\) Shear bands Analysis and Shear Moduli Calibration, *Int. Journal Solids and Structures* vol.39 pp. 3757-3776](#)

First of all, we would like to thanks the reviewers for their reading of the modified versions of this article, and for their remarks which make possible to improve the quality of our work.

Reviewer #3:

I understand the reluctance of the authors to compare model simulations with actual data but that remains my (minor?) concern: 'The examples shown no doubt represent correct implementations of the model. We have no evidence to suggest that the response is in fact appropriate for the target application of modelling granular flow.'

The question to compare the global model to experimental data is of course of extreme importance - notably to convince the engineers in the field.

On one hand, each individual part of the global model has been shown to have, independently, a good agreement with experiential datas:

- the elasto-plastic constitutive relation is not new. PLASOL is the basic constitutive relation of FEM code "LAGAMINE" and it has been clearly characterised for 15 years (Barnichon, 1998) as a simple but convenient relation essentially for practical applications, as (i) it is not associated and (ii) it includes hardening parameters. Both these ingredients are usually recognised as necessary to describe the main features of geomaterial behaviour.

- Since many years, the Bingham viscous relation is commonly used in geophysics and rheology to describe flow slides since, even if it does not take into consideration the non-linearity of viscosity, it is globally consistent with the flow of geomaterial suspensions (Coussot et Piau 1994, "On the behavior of fine mud suspensions", rheological acta).

- the solid-fluid transition criterion is defined as the second-order work criterion. This stability criterion is not new (it has been proposed by R. Hill half a century ago - Hill, 1958) and its application to geomechanics has been validated for 20 years from experimental, mathematical, analytical, and numerical (FEM and DEM) points of view in a number of papers published by several journals (IJP, IJSS, MoM, IJNAMG, GRAMA, COGE, ZAMM, and of course the present Journal of Engineering Mechanics).

According to the authors' opinion, these points are already developed in the text of the article.

On the other hand, concerning the global model (visco-elasto-plastic behaviour with a transition criterion), it can be observed that, in the case of the undrained test with a kinematic loading, the stress path after the transition differs from the response classically obtained for the granular skeleton (that is to say the response in terms of effective stress), characterized by a progressive diminution of pressure p and deviatoric stress q . It is important to recall that no hydro-mechanical coupling is defined in the code we used. As only the response of one phase is computed, the stress path before the transition does correspond to the inter-granular stresses within a soil whereas the stress path after the transition corresponds to the stresses within a granular suspension, that is to say within a homogenised material standing for the mixture of fluid and grains. The experimental stress path and the computational one cannot thus be compared, in a rigorous manner, after the transition. This is now specified in the text.

Besides, we recognized that, in such a test (undrained triaxial test with a displacement driven loading), as the deviatoric stress q decreases progressively, the transition between a solid and a fluid behaviour indeed seems to not be instantaneous. Nevertheless, according to us, such a progressive response is hardly reproducible without hydro-mechanical coupling.

Finally, even if the introduction of a hydro-mechanical coupling is the main point to improve in our modelling, the association of two constitutive relations is an efficient approach since the response captures both the initial solid behaviour and the final fluid one and makes possible a transition (even instantaneous) from one kind of behaviour to another from the material failure.

There is at least one 'unique' that should be 'unified' towards the end of the paper.

The two occurrences of 'unique' in the text have been replaced.

Reviewer #2:

The readability is seriously hampered by the English writing and style. All remarks concerning the English writing and the style have been taken into consideration.

For instance, it would be quite difficult for a reader to pick up the details of the model and implement them so as to reproduce the numerical results or to extend the model.

More details have been added, notably how the numerical implementation

is modified once the solid-to-fluid transition is enabled (paragraph 4.1.a).

The abstract does not seem to convey the above central ideas in a simple language.

The abstract has been written again to be clearer.

Furthermore, the abstract mentions about 'strong potential of the model', while the numerical results pertain to only homogeneous cases.

The previous sentence has been rephrased this way:

'... making possible a future application of this approach to gravitational flow modeling'.

"Unassociated" should be replaced with "non-associated".

This has been replaced in all the text.

Lines 26/27. What is meant by "flowing form"?

To be clearer, this expression has been replaced by: "These flows of soils have been shown ...".

Line 47. Jamming has a very specific defining in granular physics. Please define jamming more formally.

"Jamming" is a notion introduced by the physicists to characterise the transition of behaviour that can exhibit the divided matter. Indeed, depending on the loading conditions, granular material behaviour can be "solid" like (it is referred to as the "jammed state") - as the sand on a flat beach - or "fluid" like (it is referred to as the "unjammed state") - as sand avalanches on dunes. It is now specified in the introduction.

Line 58. Can the second order work be exclusively used to detect the solid-fluid transition? One could imagine a slope collapsing with eventual flow slide due to the localization of deformation over a thin layer at its base.

As explained in the text, the second-order work criterion can detect all kinds of failures due to divergence instabilities, what is including localised and diffuse modes of failure. While Rice's criterion detects only localised failure modes, second-order work criterion detects both. Rice's criterion appears to be a particular case of the second order work criterion, since when the first one is satisfied, the last one is necessarily verified (see among others Nicot and Darve 2011, "Diffuse and localized failure modes, two competing mechanisms"). Thus both failure modes are included in the present analysis.

Line 76. Flutter instabilities can be well visualized in the context of

structures such as airplane wings. Please give a practical instance when flutter instabilities occur in soils.

This is still an open, but important question in our opinion. Theoretically, there is no reason to not consider flutter instabilities in soils, which are exhibiting other kinds of material and geometrical instabilities. If so, flutter instabilities will be observed moreover for specific loading conditions.

Line 194. "stress frame" reads really weird. You mean principal stress space. There are many other instances throughout the paper where the word "frame" has been used inappropriately. Please rectify.

All the occurrences of "frame" have been replaced:

"from the failure envelope in the stress diagram "

"the pressure axis being better suited to studying the solid-like response before failure, and the time axis better adapted to describing the fluid-like response "

"Along the time axis..."

"The evolution of q is plotted along computational time (Fig. 10a) and pressure (Fig. 10b)."

"Figure 10: Stress loading path along time (a) and pressure (b)"

Line 213. "U" means union symbol. Please correct.

"U" has been actually used in this sense.

Line 234. "overstepped" should simply be replaced by "exceeded". There are other places where "overstepped" was used. Please correct.

It has been corrected as recommended.

Lines 254/255 etc... Since the integration points are moving, how does one ensure the accuracy of the numerical integration? Remember that in the original finite element scheme, the gauss points are optimally placed with corresponding weights.

The accuracy of the integration scheme obviously depends on the integration point positions. As these last are moving, the numerical weights of integration points do not take the classical values determined by the Gaussian quadrature for optimistic positions into the element.

Hence, the numerical weight must be updated at each step, according to the current location. This is now specified in the text:

"At each step, the numerical weight of each integration point is also updated in order to satisfy the conditions of the Gaussian quadrature, such that the integration is the most accurate possible for the aimed polynomial order (Moresi et al. 2003)."

Line 281, Eq. (11). Shouldn't be the " $s/2\eta$ " in Eq. (11) be replaced

with the right hand side of eq. (10)?

That is right, and the formulation is now described also once the solid-to-fluid transition is achieved (end of paragraph 4.1.a).

The viscosity ' η ' is actually an equivalent viscosity describing the fluidized material. It is certainly not constant, but it could increase or decrease with shear strain rate. Please discuss this issue in the context of mud flow as to whether one can have shear thickening or shear thinning depending on initial material strength characteristics.

It is totally right that mudflows, as most of granular suspensions, exhibit a non-linear viscosity (as already specified in introduction), and it has been added that this non-linear viscosity is often characterized by shear thinning, due to the disorganization of the granular structure.

In some very specific cases, and if the solid fraction is sufficient, the granular suspension can be characterized, on the contrary, by a shear thickening (see in Coussot&Ancey, 'Rheophysical classification of concentrated suspensions and granular pastes', Physical Review, 1999).

Line 296/297. The plasticity scheme seems to be explicit. Does this affect numerical convergence?

The plasticity scheme is indeed explicit since the 'plasticity force term' f_t is quantity known from the previous step. To have a good convergence and accurate results, the loading increments have to be small enough. In practice, we made tests with the code to determine the maximum loading step that could be applied to have sufficiently accurate results. That is now mentioned in the text.

Line 302. "A unit square of soil" sounds strange. You mean a unit square representative elementary volume of soil.

This has been change in the text.

Line 305. "Keeping focused on flowslide hazards" also sounds strange. You mean "relating to flowslide hazards"

This has been replaced in the text.

Line 312. "extensive" ... you mean "extension".

This has been corrected in the text for a better understanding: "Compression strain in vertical direction z and extension strain in horizontal direction x ".

Lines 319/320. Do the magnitudes of stress and strain increments influence the calculation of the second order work? If so, how small these magnitudes should be?

The magnitude of stress and strain increments does change the value of

the second-order work but not the value of its normalized expression (see eq. 3), and in any case the change of sign will happen at the same stage of the loading path.

Line 322. "pressure" should be mean effective stress.

It has been considered in all the text that the pressure is the mean stress (it is now recalled in paragraph 4.1.b). Besides, we consider that using the expression 'effective stress' is a little bit misleading since the model only takes into account one phase, without hydro-mechanical coupling.

Nevertheless, it is recalled later in the text that the pressure during the computation has a different meaning before and after the transition. Indeed, it stands first for the intergranular pressure, whereas after the transition it stands for the pressure in a homogenized material (the granular suspension) standing for the grains-water mixture. This is due to the absence of coupling.

Line 333. Please explain why the mean effective stress 'p' remains constant as the deviatoric stress drops to zero at peak condition.

The reason why p remains constant after at the transition is already explained in the text (paragraph 4.1.c): 'no viscous volumetric strains can take place since the Bingham relation is purely deviatoric, and elastic volumetric strains are null because the loading is isochoric'. On the contrary, deviatoric stresses are relaxed by viscosity.

Line 350/353. Please rephrase for more clarity.

It has been done: 'Nonetheless, the triggering of mudflows in the field is generally linked to mixed loading conditions (stress and displacement ones) with an important role of the stress conditions (water table rising for instance). The total constraint of the kinematic, which prevents the effective failure development, is thus unlikely.'

Section 4.3. Please explain "jamming" of flow and the piece of physics that leads to it in the model.

This term is now explained in the introduction. It is simply taken into account in the model by mean of the stress threshold so.