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HAL Id: hal-00586416
https://hal.univ-smb.fr/hal-00586416
Submitted on 8 Jan 2013

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Supervisory Control based Fuzzy Interval Arithmetic Applied for Discrete Scheduling of Manufacturing Systems

K. Tamani, R. Boukezzoula and G. Habchi

Abstract — This paper considers the modelling and designing of a production-flow scheduler based on fuzzy interval system. Particularly, the supervisory control is built according to the satisfaction degree of conflicting objectives which are quantified by fuzzy intervals. The control system aims at adjusting the machine’s production rates in such a way that satisfies the demand while maintaining the overall performances within acceptable limits. At the shop-floor level, the actual dispatching times are determined from the continuous production rates through a sampling procedure. A decision for the actual part to be processed is taken using some criterions which represent a measure of the job’s priority. A case study demonstrates the efficiency of the proposed control approach.

I. INTRODUCTION

The scheduling of job-shop manufacturing systems with flexible machines and producing multiple part types has been studied by many approaches. The most developed ones have been enumerative algorithms that provide exact solutions either by means of elaborate and sophisticated mathematical constructs, such as linear [8] and constraint programming [11]. However, the limitations of the enumerative techniques have led to suboptimal approximation methods using simulation [6]. Furthermore, in the case of incomplete or imprecise data knowledge, some solutions for scheduling problems have been provided according to artificial intelligence techniques, including neural networks, fuzzy logic and evolutionary algorithm [11][4][15].

The research reported in this paper is based on this last idea where a fuzzy system is used in a two levels control structure for discrete scheduling problems. Indeed, given a job-shop manufacturing system, this research attempts to address, at the shop-floor level, the discrete dispatching of the machine production rates computed at the flow control level. In this case, the proposed approach uses continuous control theory [5][16] and artificial intelligence techniques for production flow regulation of realistic (in terms of modelling assumptions) manufacturing systems [10][13][15].

In our previous work, a production flow control strategy based on fuzzy interval arithmetic for multi-objective optimization has been developed. Indeed, the supervisor combines multiple and possibly conflicting objectives such that a best compromise can be achieved between them. In this case, the overall objectives are quantified by fuzzy intervals since they are specified as imprecise and uncertain information.

However, the provided control actions (production rates) are continuous time expression while the production operations are of discrete nature. Thus, in order to deal with a scheduling problem, there is need to manage the transient from the flow control (continuous) to the shop-floor level (discrete). For this purpose, the developed scheduler is based on sampling procedure which translates the continuous-time production rates, computed at the flow control level, to a series of loading times at the shop-floor level. In this case, the actual loading part is taken according to the route priority.

The rest of the paper is organized as follows. Section 2 describes the continuous-flow approximation to model the discrete flow of parts in manufacturing systems. The continuous-flow control methodology is presented in section 3. Section 4 introduces the sampling and dispatching procedure for discrete real-time scheduling of part types at shop-floor level. Section 5 illustrates the scenario and the experimental results for re-entrant and multi-product real manufacturing system. Finally, concluding remarks are given in section 6.

II. CONTINUOUS-FLOW DYNAMIC MODEL

The manufacturing system can be viewed at the shop-floor level as a network of a finite number of machines and buffers. Thus, when considering a system composed of $N$ machines $M_i (i=1,..., N)$, it may be decomposed into $N$ basic production modules $PM(i)$. Each one is composed of a machine $M_i$ and its sets of upstream and downstream buffers. For instance, in the case of a transfer line (Fig. 1), the production module can be defined as $PM(i) = \{B_{i-1}, M_i, B_i\}$.

![Fig. 1. Transfer line](image)

For the sake of simplicity, the developments are given for a single-part-type system depicted in Fig. 1. The level of buffer $B_i$ is given by the variable $x_i$, collecting
continuously the products coming from machine \( M_i \) and feeding machine \( M_{i+1} \). The machines are supposed reliable. The production rate of machine \( M_i \) at time \( t \) is denoted by \( u_i(t) \) and the required processing time, noted \( r_i \), is supposed known and deterministic. Thus, the increasing rate of buffer \( B_i \) is a function of the production rate \( u_i \) of the feeding machine \( M_i \). The decreasing of buffer level \( x_i \) is in relation with the processing rate \( u_{i+1} \) of the downstream machine \( M_{i+1} \). Therefore, by aggregating the increasing and decreasing rates, the dynamic model of the evolution of buffer level (production-flow) \( x_i \) is given by:

\[
\dot{x}_i(t) = u_i(t) - u_{i+1}(t), \quad 0 \leq x_i(t) \leq x_i^{\text{max}} \tag{1}
\]

This dynamic equation represents the basis of the continuous-flow model used in simulation. The restriction in (1) concerns the inability of buffer \( x_i \) to increase its content while the capacity bound \( x_i^{\text{max}} \) is reached.

Let us define the fraction of the capacity of \( M_i \) devoted for processing at time \( t \) as follow:

\[
r_i(t) = \frac{u_i(t)}{u_i^{\text{max}}}, \quad 0 \leq r_i(t) \leq 1 \tag{2}
\]

where \( u_i^{\text{max}} = 1/r_i \), and \( 0 \leq u_i(t) \leq u_i^{\text{max}} \). In this paper, \( r_i(t) \) represents the control variable to be defined that adjusts the production rate between zero and its maximum. Further, in order to track the demand at each production means, the production surplus \( s_i \) (tracking error), defining the difference between the cumulative production (performance measure) at this means (denoted \( y_i \)), and the demand, is taken into account in the design of the closed loop control system.

### III. CONTINUOUS-FLOW CONTROL METHODOLOGY

Given a manufacturing system represented by the production-flow dynamic model (1), the control objective is to adjust the production rates, through an appropriate capacity allocation policy, in such a way to reach a predefined required production while keeping all overall performance measures within their acceptable values [13]. For this purposes, the continuous-flow control methodology of two levels has been developed with a set of distributed fuzzy controllers at the lower level and a supervisory controller at the higher level. This section recalls the flow control methodology principles with focuses on the supervisory control strategy

#### A. Distributed Fuzzy Control For Machine’s Capacity Allocation

To make clear how the distributed fuzzy control strategy is designed, the basic idea is illustrated through the elementary transformation module \( PM(i) \). The control objective is to track the demand while keeping the upstream and downstream buffers \( B_{i-1} \) and \( B_i \) of \( M_i \) neither full nor empty. This is achieved by allocating an optimised machine capacity to production at each instant according the following statements:

- If the surplus level is satisfying, then try to prevent starving or blocking by increasing or decreasing the production rate of the machine.
- If the surplus level indicates backlog or excess inventory, then produce respectively with the maximum or zero rate.

In this case, the control law is determined on the basis of the expert knowledge, where a fuzzy system, constituting a controller, has been used. Indeed, the fuzzy controller \( FC(i) \) has been formalized by using a Takagi-Sugeno system [12] as follows:

\[
R_{ji}^{(i_1,i_2,i_3)}: \quad \text{if } x_{i-1} \text{ is } X_{i}^{(j_1)} \text{ and } x_i \text{ is } X_{i}^{(j_2)} \text{ and } s_i \text{ is } S_{i}^{(j_3)}, \quad \text{then } r_i = \phi_{ji}^{(i_1,i_2,i_3)} \tag{3}
\]

where:

- \( X_{i-1}^{(j_1)}, X_i^{(j_2)} \) and \( S_i^{(j_3)} \) correspond to the \( i_j \) linguistic term of the input variables \( x_{i-1}, x_i, s_i \) respectively from the sets \( X_{i-1} = \{ \text{Empty, Almost Empty, Normal, Almost Full, Full} \} \) and \( S_i = \{ \text{Backlog, Normal, Inventory} \} \)
- \( \phi_{ji}^{(i_1,i_2,i_3)} \) is the real value involved in the rule conclusion indexed by \( (i_1, i_2, i_3) \) that gives the fraction of capacity devoted to processing.

Fig. 2 illustrates the fuzzy control structure \( FC(i) \) for a transformation operation.

![Fuzzy Controller](image)

**Fig. 2. The fuzzy control structure**

The output variable of the controller represents a weighting factor \( r_i(t) \) to range the production rate of \( PM(i) \) between zero and its maximum \( u_i^{\text{max}} \). The complete rulebase for a fuzzy controller of a transformation module is given in [13].

Finally, when considering a general manufacturing system composed of \( N \) modules, the fuzzy control design detailed above has been deployed for each ones, which leads to a distributed fuzzy control (DFC) structure.

#### B. Supervisory Based Fuzzy Interval Arithmetic

In fully distributed control systems, global optimization is hard to obtain due to the “myopic behaviour” of distributed control systems. In order to deal with myopic behaviour, it is necessary to define a kind of “global optimizing mechanism” (GOM) [14].

There are several ways to integrate GOM into distributed control systems. In our case, global specifications are imposed within which global performance level must be maintained. Indeed, given a set...
of performance indicators P = {P1, ..., Pl} with associated objectives Pobj = {Pobj1, ..., Pobjl}, the supervisory control aims at reinforcing the local control action through an additive component in order to compensate the deviations of performance measures from their objectives. The key idea of the supervision function resides in: (i) the fuzzy intervals representation of the objectives and (ii) the combination mechanism based on the fuzzy interval arithmetic.

The second step is performed in the same way by considering the precise performance indicator measures according to the precise operator [3]. Finally, at the third step, the resulted satisfaction degree (the α-cut) is used to determine the additive component (supervisory control action), denoted rci, under the constraint of the local control rci.

At the first step, a trapezoidal fuzzy interval, denoted by P T obj, has been used to represent the objective associated to the performance indicator P T as illustrated in Fig. 3. The fuzzy interval is formalized by the left and right profiles denoted (P T obj)l and (P T obj)r, respectively [7]. In the case of trapezoidal shape, they are defined by:

\[
\begin{align*}
(P_T^{\text{obj}})_l(\alpha) &= (1-\alpha) \cdot a_l^{\text{obj}} + \alpha \cdot b_l^{\text{obj}} \\
(P_T^{\text{obj}})_r(\alpha) &= (1-\alpha) \cdot a_r^{\text{obj}} + \alpha \cdot c_r^{\text{obj}}
\end{align*}
\]  

(4)

Thus, given the fuzzy intervals of the objectives P obj and their performance measures P, the principle of the proposed supervision mechanism is summarized on the following three steps:

1) Combine the objectives P obj = {Pobj1, ..., Pobjl} through an uncertain operator Ψ, since they are defined by fuzzy intervals. The combined objective is a fuzzy interval denoted P T obj.

2) Combine the performance indicator measures P = {P1, ..., Pl} using the precise version of the operator Ψ, denoted ψ. The combined measure is denoted P T.

3) Evaluate the resulted precise measure P T with regard to the combined fuzzy objective P T obj. The result represents the satisfaction degree of the combined objective (the α-cut).

At the first step, the arithmetic operations on fuzzy intervals are used according to the profiles representation (4). In this case, the uncertain operator can be implemented [2]. For instance, when using the weighted mean operator, the resulted fuzzy interval is expressed as follows:

\[
P_T^{\text{obj}} = \Psi[P_T^{\text{obj}}(\alpha), ..., P_T^{\text{obj}}(\alpha)] = \sum_{i=1}^{\Omega} w_i \cdot P_T^{\text{obj}}(\alpha)
\]  

(5)

where \(\Omega\) is the fuzzy addition between fuzzy intervals such that: \((P_T^{\text{obj}} \oplus P_T^{\text{obj}})(\alpha) = [(P_T^{\text{obj}})(\alpha) + (P_T^{\text{obj}})(\alpha)], (P_T^{\text{obj}})(\alpha) + (P_T^{\text{obj}})(\alpha)]\).

Fig. 3. Trapezoidal fuzzy interval representation

Fig. 4 shows the domain values of rci, which are encapsulated within a triangular fuzzy interval RT with the support \(R_T(0) = [-r_{ci}, 1-r_{ci}]\) and the kernel \(R_T(1) = 0\).

For practical implementation, the supervisory control is determined according to the following statements:

- If P T evolves within the kernel of P T obj, the system behaviour is in normal mode. This means that the satisfaction degree of the objective is total (α = 1). In this case, the supervisor does not provide additive component (rci(t) = 0).

- If P T evolves outside the support of P T obj, a fully degraded operating mode is detected. The objective in this case is totally unsatisfied (α = 0), and the supervisory action is given by:

\[
rci(t) = \Delta - r_{ci}(t) \quad \text{with} \quad \Delta = \left\{ \begin{array}{ll}
1 & \text{if} \ P_T \leq a_{T,\text{obj}} \\
0 & \text{if} \ P_T \geq a_{T,\text{obj}}
\end{array} \right.
\]  

(6)

It consists in either allocate the maximum remaining capacity (Δ = 1) or stop the productivity of the module (Δ = 0).

- If P T evolves in the switching modes, the corresponding α-cut of the fuzzy interval P T obj is used to determine the supervisory control. Indeed, whether P T evolves on the left or right profile, the α-cut level is given by the reverse of the corresponding profile function. That is, when P T evolves on the left profile, the supervisory control is given as:

\[
r_{ci}(t) = (1-\alpha) \cdot (1-r_{ci}(t)) \quad \text{with} \quad \alpha = \left(\frac{P_T^{\text{obj}}}{P_T}\right)^1
\]  

(7)

In this case, the action attempts to allocate a fraction of the remaining capacity.
When \( P_1 \) evolves on the right profile, the supervisory action attempts to reduce the productivity of the controlled module as follows:

\[
    r_{i}(t) = (1 - \alpha) \cdot (-r_{i}(t)) \quad \text{with} \quad \alpha = (P_{1}^{\text{obj}})^{(P_{1}^{\text{obj}})^{-1}}
\]  

(8)

The functions (7) and (8) represent respectively the right and left profiles of a triangular fuzzy interval \( R_{i} \) of the supervisory control domain (Fig. 4).

Finally, according to the local control given by the fuzzy controller and the supervisory control, the production rate is adjusted as follow:

\[
    u_{i}(t) = (r_{i}(t) + r_{i}(t)) \cdot u_{i}^{\text{max}} = r_{i}(t) \cdot u_{i}^{\text{max}}
\]  

(9)

IV. DISCRETE REAL-TIME SCHEDULING METHODOLOGY

In our case, the scheduling problem involves two types of decisions at this level:

- to determine the loading times of actual parts and
- to resolve the conflicts in the case of multiple-part-type systems.

For the first decision, a dispatching policy has to be used in order to determine the loading times of actual parts. Indeed, since the machine operation frequency is equivalent to the time between two successive machine loads, at a certain time, the sampled value is held constant during a time interval equal to its reverse. The holding period includes the operation and the idle times. Thus, the continuous time production rate is translated to a piecewise constant function as shown in Fig. 5.

![Fig. 5. Continuous production rate discretisation](image)

Using this definition, as the production rate evolves between 0 and \( u_{i}^{\text{max}} \), the lower bound correspond to an infinite idling time (no production) while the upper bound corresponds to the operation time (no idle time). For practical use, in order to limit the idle period when the production rate is too low, the lower bound is chosen equal to 50% of its maximum.

For the case of a multiple-part-type system, a machine \( M_i \) may operates on different part types \( j \) such that \( j \in Q(i) \), where \( Q(i) \) is the set of part types to be processed on \( M_i \) and its cardinality is equal to \( J(i) \). Each of them may involves \( K_j \) \((k = 1, ..., K_j)\) different operations (case of re-entrant flow if \( K_j > 1 \)). In this case, the original machine \( M_i \) is virtually divided into \( N(i) = \sum_{j \in Q(i)}K_j \) single-part-type sub-machines \( m_{i,j} \). Only one submachine is allowed to work at a time.

Thus, for the second decision, the criterion value representing the route priority measurement is derived on the basis of the control input values; the surplus performances (local and final) and the order of the operation in the case of re-entrant flow. The part to be loaded is the one with the largest criterion value.

The proposed criterion value for each submachine \( m_{i,j} \) of a certain multiple-part-type machine \( M_i \) is given by the following weighting sum:

\[
    J_{i,j} = \sum_{l=1}^{4} \pi_i \cdot g(c_{i,j,l})
\]  

(10)

where:

- \( c_{i,j,l} \) is the sampled value of the computed production rate \( \hat{u}_{i,j,l} \) of the submachine \( m_{i,j} \)
- \( c_{i,j}^{o} \) is its corresponding local surplus such that \( c_{i,j}^{o} = \max\{0, -s_{i,j} \} \)
- \( c_{i,j}^{l} \) is the finished surplus level \( s_{i,j} \) of the part-type \( j \) with \( O(j) \) is the last submachine of its route,
- \( c_{i,j}^{g} \) is the order \( k \) in which the part of type \( j \) visits the machine \( M_i \).

In the criterion definition above, \( g(.) \) is a positive monotonically increasing non-linear function, with \( g(0) = 0 \) and \( g(c^{o}_{i,j}) = 1 \) for \( c^{o}_{i,j} \to \infty \). This function can be closely approximated by sigmoidals of the form: \( g(c^{o}_{i,j}) = 1/(1 + \exp(-c^{o}_{i,j})) \) \[10\]. According to the measures of \( c_{i,j}^{o}(l = 1, ..., 4) \), this function gives the maximum value for the route (submachine) which presents the highest calculated production rate, the larger backlog (negative local and final surpluses) and the latest operation in the case of re-entrant flow. The values of \( c_{i,j}^{o}, c_{i,j}^{l} \) and \( c_{i,j}^{g} \) lead to a criterion with a local scope, while \( c_{i,j}^{g} \) introduces global insight of the state of the actual route. The parameters \( \pi_i \) are the weighting factors to be chosen according to the importance of each element \( c_{i,j}^{o} \). The following algorithm summarizes a practical implementation of the discrete dispatching procedure:

**Inputs**
- \( u_{i} \in R^{(j)} \), \( s_{i} \in Z^{(j)} \), \( s_{O,j} \in Z^{(j)} \) with \( O_{j} = \{O(j) \mid j = 1, \ldots, J(i)\} \).

**Outputs**
- The selected submachine \( m_{i,j} \) with its discrete production rate \( \hat{u}_{i,j} \), loading time \( t_{s_{i,j}} \) and holding time interval \( \hat{u}_{i,j}^{-1} \).

**Begin**
1. For all not idle submachines
   - Calculate \( J_{i,j} \) according to (10).
   - Select the submachine \( m_{i,j} \) having the highest \( J_{i,j} \).

**Endfor**
2. The production rate \( u_{i,j} \) of the selected submachine is sampled at a time \( t_{s_{i,j}} \) \((n = 1, 2, \ldots)\) corresponding to the loading instant. A time interval equal to the
end

The number of operations for the second machine is equal to 7 instead of 6, since it serves part type 1 twice (re-entrant flow). The same holds for machine $M_2$. Furthermore, raw materials arrive in the cell at a rate of 0.03 parts per time unit, implying that for each route a raw material arrives every 34 time units.

Based on the workload of the cell bottleneck machine, i.e., machine $M_2$; the authors in [10] define a lower bound for the achievement of the production demand (makespan) which serves as the reference for comparison purposes. Specifically, the machine $M_2$ (7 submachines) must process 20 parts requiring $7 \times 20 \times 6 = 840$ time units. As the first raw material arrives in the cell at time 34, a lower bound of 874 time units has been derived.

In order to evaluate the effect of the supervision, the proposed methodology is simulated in both cases: without supervision (distributed fuzzy control – DFC) and with supervisory control (supervisory fuzzy control – SFC). When integrating the supervisory control, the overall performance indicators of the average and the instantaneous finished surplus, and the total production cost are used. This latter is given by:

Total Cost $= c_{wip} \cdot WIP + c_{inv} \cdot INV + c_{bck} \cdot BCK + c_{lt} \cdot LT \quad (11)$

The first two terms of (11) represent the cost measures of storing parts in buffers. Specifically, measures for the work-in-process and inventory costs are provided by means of the average integral of the intermediate and output buffers respectively. The two last terms of (11) are concerned with the average backlogging costs and the average lead time costs.

The cost units $c_{wip}$, $c_{inv}$, $c_{bck}$, $c_{lt}$ for all the performance measures in (11) are taken equal to 1 for simplicity. The measure of the total cost in the supervisor is chosen as the reverse of (11). The associated objectives, expressed by fuzzy intervals through the profile functions (4), are fixed, for the surplus performances, as:

$P^{obj}_1 = P^{obj}_2 = [-3 + 2\alpha, 3 - 2\alpha]$, and for the total production cost performance as:

$P^{obj}_3 = [0.01 + 0.09\alpha, 1.1 - 0.1\alpha]$. When using the arithmetic mean operator (5), the resulted combined interval is: $P^{obj}_T = [-1.996 + 1.363\alpha, 2.366 - 1.366\alpha]$. The parameters $\alpha$ of the criterion (12) are taken respectively equal to $0.4$, $0.25$, $0.25$, $0.1$.

The obtained results are compared to the conventional FIFO strategy and those provided in [10], and are summarized in Table II for the case of reliable machines. In this case, the SFC methodology achieves the demand with the exact calculated lower bound and the utilisation rate of the bottleneck machine ($M_2$), which is approximately 96%, is improved in comparison to the rate reached with a DFC methodology (Table III).
### TABLE III

<table>
<thead>
<tr>
<th>Machine utilisation rates (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Methodology</strong></td>
</tr>
<tr>
<td>SFC</td>
</tr>
<tr>
<td>DFC</td>
</tr>
<tr>
<td>FIFO</td>
</tr>
</tbody>
</table>

### VI. CONCLUSION

In this paper, the potential application of the production-flow control for discrete scheduling of a manufacturing cell is investigated. The production-flow control methodology is based on arithmetic fuzzy interval to build a decision according to the satisfaction degree of the conflicting objectives quantified by fuzzy intervals. At the shop-floor level, the scheduling problem is addressed in two steps. The first step performs the transition from a computed continuous control to a discrete dispatching control through a sampling procedure. The second step deals with the conflicts of multiple routes by using some criterion representing a measure of the priority.

The only uncertainties considered in this paper are the overall objectives quantification. An important open issue is the robustness of the methodology when other forms of uncertainty are present, such as machine failures, random arrival, setup times etc.

### TABLE II

**SIMULATION RESULTS WITHOUT MACHINE FAILURES**

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Makespan</th>
<th>Avg. WIP</th>
<th>Avg. inventory</th>
<th>Avg. backlog</th>
<th>Avg. lead time</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFC</td>
<td>874</td>
<td>0.894</td>
<td>9.393</td>
<td>2.978</td>
<td>65.46</td>
<td>78.73</td>
</tr>
<tr>
<td>DFC</td>
<td>1007</td>
<td>1.636</td>
<td>8.516</td>
<td>5.604</td>
<td>74.68</td>
<td>91.345</td>
</tr>
<tr>
<td>FIFO</td>
<td>877</td>
<td>1.016</td>
<td>10.045</td>
<td>0.00466</td>
<td>215.96</td>
<td>224.64</td>
</tr>
<tr>
<td>DNN</td>
<td>963</td>
<td>0.506</td>
<td>8.17</td>
<td>0.00262</td>
<td>142.769</td>
<td>154.96</td>
</tr>
<tr>
<td>CAF</td>
<td>1044</td>
<td>1.347</td>
<td>10.848</td>
<td>0.00214</td>
<td>125.72</td>
<td>138.34</td>
</tr>
<tr>
<td>CLB</td>
<td>1083</td>
<td>1.149</td>
<td>11.468</td>
<td>0.00214</td>
<td>125.72</td>
<td>138.34</td>
</tr>
</tbody>
</table>

### REFERENCES


